

An alternative cosmological model compatible with the Λ CDM model today.

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Abstract

The theory of quantum mechanics and the theory of general relativity always discuss the initial conditions of the universe and its evolution. We will try to add some simple considerations to this question through a simple cosmological alternative model based on the Hubble time, Planck mass flow rate and a variable coefficient α_H . We find the parameters obtained by the Planck 2018 results with the Planck mass flow rate and Hubble time.

Keywords : cosmology, origin of universe, dark energy, Hubble constant, Planck mass flow rate, evolution of universe, quantum mechanics, general relativity, antimatter, Big bang.

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Introduction

The Λ CDM model based on Einstein's theory of general relativity and on observations is today the most satisfactory theoretical proposal to describe the universe. On the other hand, no quantum description of the universe is now in consensus. We can note that the Planck mass flux is both a relativistic quantity (c^3/G) and a quantum quantity ($m_{\text{Pl}}/t_{\text{Pl}}$). We will use this quantity associated to the Hubble time to propose a quasi complete alternative theoretical framework, relativistic and quantum, of the universe. This alternative theoretical framework, which follows from the Λ CDM model finds values consistent with the results of the Planck 2018 measurements, tries to explain what dark energy is. It proposes an explanation for the vacuum catastrophe. Finally it recovers the cosmological diffuse background temperature determined by the WMAP satellite with the Planck 2018 results in a simple and easily affordable cosmological model.

A) A toy cosmological model compatible with the Λ CDM model after the decoupling.

It seems possible to obtain the total mass of the universe from the Λ CDM model otherwise. This could eventually lead to the development of a simple toy cosmological model unknown to the author, built around the Hubble constant, the Hubble time, $t_H = 1/H$, the Planck mass flow and a variable coefficient α_H .

$\alpha_H =$ **radius of the observable universe** (from calculation of the Λ CDM model for example) **divided by the Hubble radius** at time t_H for a flat universe ;

$$\alpha_H = \frac{c}{H_0} \int_{a=0}^{a=1} \frac{da}{a^2 \sqrt{\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda}} / \frac{c}{H_0} \quad (\text{Equation 1})$$

where a is the scale factor, c is the speed of light, $H_0 = 67,4 \pm 0,5 \text{ km s}^{-1} \text{ Mpc}^{-1}$, is the Hubble parameter measured today^[1], the Ω_i are the density parameters of the standard cosmological model, i.e. the Λ CDM model, measured today^[1].

$$\alpha_H = \int_{a=0}^{a=1} \frac{da}{a^2 \sqrt{\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda}} \quad (\text{Equation 2})$$

$\delta = \frac{c^3}{G} = \frac{m_{Pl}}{t_{Pl}}$ is the Planck mass flow rate.

$t_{H_0} = \frac{1}{H_0}$ is the Hubble time ($\approx 4,578 \cdot 10^{17} \text{ s} = 14,51$ billion light years today)

R_{H_0} is the Hubble radius.

$$R_{H_0} = \frac{c}{H_0} = c t_{H_0} \quad (\text{Equation 3})$$

The increase of the "total mass", M_{H_0} , of the universe at Hubble radius in the sense of the Λ CDM model is determined for a flat universe by the relation :

$$M_{H_0} = \frac{3}{8\pi G} \frac{4\pi}{3} (c t_{H_0})^3 \quad (\text{Equation 4})$$

$$M_{H_0} = \frac{1}{2} \frac{c^3}{G} t_{H_0} \quad (\text{Equation 5})$$

$$M_{H_0} = \frac{1}{2} \frac{m_{Pl}}{t_{Pl}} t_{H_0} \quad (\text{Equation 6})$$

$$M_{H_0} = \frac{1}{2} \delta t_{H_0} \quad (\text{Equation 7})$$

The mass of the observable universe in the sense of the Λ CDM model is :

$$M_{H_0} \alpha_{H_0}^3 = \frac{1}{2} \delta t_{H_0} \alpha_{H_0}^3 \quad (\text{Equation 8})$$

$\alpha_{H_0} \approx 4.399 \cdot 10^{26} \text{ m} / 1.372 \cdot 10^{26} \text{ m} \approx 3.175$ today if $H_0 = 67,4 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_m = 0,315$ and $\Omega_\Lambda = 0,685$ ^[1].

$$M_{H_0} \alpha_{H_0}^3 \approx 2,959 \cdot 10^{54} \text{ kg} \quad (\text{Equation 9})$$

in other words, the "total mass" of the observable universe Λ CDM measured today. ($e=mc^2$)

B) Value of α_H before the decoupling in the cosmological toy model and consequences.

The author hypothesises that, before the decoupling, the radius of the observable universe was equal to the Hubble radius. The ratio α_{H_0} was then equal to 1.

B.1) Thus, the mass of the universe at $t_{H0} = \text{Planck time}$ is determined by :

$$M_{H_{t_{Pl}} = \frac{1}{2} \frac{m_{Pl}}{t_{Pl}} t_{Pl}} \quad (\text{Equation 10})$$

$$M_{H_{t_{Pl}} = \frac{1}{2} m_{Pl}} \quad (\text{Equation 11})$$

This can be verified with the thermal energy :

$$E_{Th} = \frac{1}{2} m_{Pl} c^2 = \frac{1}{2} k_B T_{Pl} \quad (\text{Equation 12})$$

where k_B is the Boltzmann constant, with one degree of freedom assumed for the singularity and T_{Pl} the Planck temperature.

B.2) Mass of the universe at Hubble radius in this alternative cosmological model.

Starting from a "Planck time grain mass", the singularity of the Big Bang model, at the beginning of the time of the universe, . Then by making the assumption that for each unit of Planck time that passes, a corresponding mass "Planck time grain mass" is added to the mass of the universe. In our toy cosmological model, the "total mass" (energy) of the universe at the Hubble radius, before and after the decoupling, at time t_{H0} , grows simply by following the summation :

$$M_{H0} = \sum_{i=0}^{t_H/t_p} i \frac{m_{Pl}}{2} \quad (\text{Equation 13})$$

i.e.

$$M_{H0} = \frac{1}{2} \frac{m_{Pl}}{t_{Pl}} t_{H0} \quad (\text{Equation 14})$$

$$M_{H0} = \frac{1}{2} \frac{c^3}{G} t_{H0} \quad (\text{Equation 15})$$

t_{H0} is the Hubble time. $H_0 = 67,4 \text{ km/s/Mpc}^{[1]}$, $t_{H0} = 4,578 \cdot 10^{17}$ seconds today, so $M_{H0} \approx 9,241 \cdot 10^{52} \text{ kg}$

Note : ... and with datas of §2 we have Eq.9, $M_{H0} \alpha_{H0}^3 \approx 2,95910^{54} \text{ kg}$

with $t_H = 1/H$, so the Hubble radius in this toy universe is the same as the Hubble radius in the Λ CDM model.

This is valid, without recourse to cosmic inflation, from Planck time to the Hubble radius of the universe at the time of decoupling in the standard model (377 700 years) but also beyond. This is made possible by writing the "total mass" and the Hubble radius as sigma summations. This also has the consequence of limiting quantum phenomena in the universe to dimensions of the order of Planck units between t_H and $t_H + t_{Pl}$.

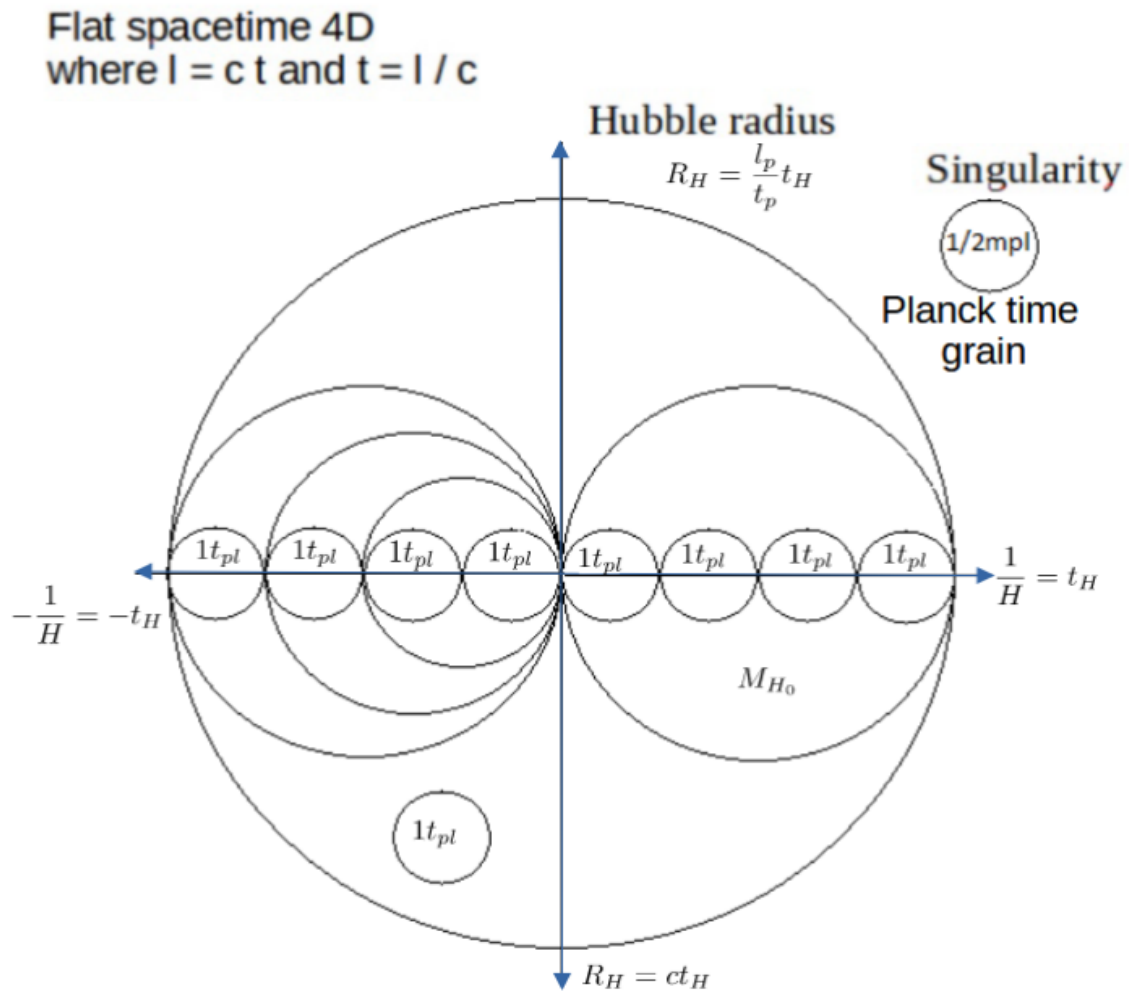


Figure 1: Hubble sphere

C) Proposal of determination of the cosmological constant in this toy cosmological model.

C.1) The Hubble sphere seen as a black hole.

Figure 1 shows that the observation of the Hubble sphere is always done in a given direction whether along the axes t_H or along the axes R_H . When we look in the opposite direction to the Earth, we observe a universe with the same characteristics, namely a Hubble universe whose mass increases as a function of t_H . This toy model is therefore by construction isotropic, i.e. identical whatever the direction of observation. It is also homogeneous on a large scale by construction, i.e. for any considered time interval t_{Pl} , it contains a Planck half-mass. For a large number of Planck

half-masses, the latter agglomerate to form stars first under the effect of dark matter and then under the gravitational effect of the Planck half-masses agglomerated under the effect of dark matter.

In other words, there is always an observational bias that makes a part of the Hubble Universe not available to our observation, but it is there. We have therefore a Hubble universe of mass $\rho_c V_{H0}$ composed of two mini Hubble spheres that could have a radius of $R_{H0} = c t_{H0}$ and a mass

$M_{H0} = \frac{1}{2} \frac{c^3}{G} t_{H0}$ because $R_{H0} = c t_{H0}$ is proportional to M_{H0} . These two mini Hubble spheres "touch" each other at the observer's original location and time. The observer sees only one of these mini-spheres at any given time

The invariant gravitational force that attracts these masses of the two mini Hubble spheres is therefore $FM_{H_0}^{\pm}$:

$$FM_{H_0}^{\pm} = \frac{GM_{H_0}^+ M_{H_0}^-}{R_{H_0}^2} \quad (\text{Equation 16})$$

$$FM_{H_0}^{\pm} = \frac{c^4}{4G} \quad (\text{Equation 17})$$

$$FM_{H_0}^{\pm} = \frac{F_{Pl}}{4} \quad (\text{Equation 18})$$

where F_{Pl} is Planck's force where N is the Newton.

$$FM_{H_0}^{\pm} = 3,02564 \cdot 10^{43} \text{ N} \quad (\text{Equation 19})$$

The Planck force characterizes a property of space-time according to Barrow and Gibbons^[2]. In general relativity, the limiting value it represents does not correspond to the Planck unit, but to the reduced Planck unit, where G is replaced by $4G$. The resulting reduced Planck force is four times weaker and is equal to Eq.16 to Eq.19 . This is a maximum limit in general relativity, attainable only at the horizon of a black hole. As the radius of a Schwarzschild black hole R_s is also its horizon R_h where $R_s = R_h = R_{H0}$ it is permissible to assimilate the Hubble universe to a Schwarzschild black hole. Its barycenter is the center of the Hubble sphere.

$$R_h = \frac{2GM_{H_0}}{c^2} \quad (\text{Equation 20})$$

Considering the Hubble sphere as a Schwarzschild black hole will be essential in a following paragraph to theorize the temperature of the cosmic microwave background, i.e. the CMB.

The two mini Hubble spheres can also be two complete Hubble spheres. We will simply note that the possibility of a double universe with two opposite time arrows proposed by the Soviet physicist Andrei Sakarov in 1967 is taken up here. But that as far as the Hubble sphere is concerned, the $M_{H_0}^+$ and $M_{H_0}^-$ universes are included in the same universe. The ideas that follow from Andrei Sakarov's hypothesis should be re-examined according to the author, especially to account for dark matter and dark energy. The hypothesis of Andrei Sakarov has given rise to few scientific works. Among the scientists who have worked on his hypothesis are Nathan Rosen, Jean Pierre Pettitt, Gabriel Chardin, Michael Boris Green, John Henry Schwarz, Abdus Salam (Nobel Prize in Physics in 1979), or Sabine Hossenfelder. Finally we note by examining two mini Hubble spheres or two

complete Hubble spheres, that $M_{H_0}^+$ and $M_{H_0}^-$ can be seen as a mass of matter and a mass of antimatter.

C.2) Proposal of determination of the cosmological constant.

We try here to treat the dark energy. In classical mechanics the gravitational interaction between two masses is instantaneous but in general relativity this interaction cannot be faster than the speed of light c . We will use this property of the theory of general relativity to propose a value of the cosmological constant. However we will encounter a problem of dimensional coherence that we will circumvent in a very debatable way for lack of a better explanation for the moment. The value which is questionable from the dimensional point of view is however in agreement with the Planck 2018 results as we will highlight later.

Since the velocity of the gravitational interaction $FM_{H_0}^\pm$ between $M_{H_0}^+$ and $M_{H_0}^-$ is limited to c we assume that the power of $FM_{H_0}^\pm$ is $PM_{H_0}^\pm$ Watts such that:

(Equation 21)

$$PM_{H_0}^\pm = 9,0706 \cdot 10^{51} \text{ kg} \cdot \text{m}^2 \cdot \text{s}^{-3} \quad (\text{Equation 22})$$

We will look for what could balance this power. As we have already used the opposite with $M_{H_0}^+$ and $M_{H_0}^-$ to find $FM_{H_0}^\pm$, this time we will use the inverse of $PM_{H_0}^\pm$ to get the neutrality equal to 1 of the mathematical operation:

$$\frac{1}{PM_{H_0}^\pm} = 1,1025 \cdot 10^{-52} \text{ kg}^{-1} \cdot \text{m}^{-2} \cdot \text{s}^3 \quad (\text{Equation 23})$$

The Watt is also the measure of energy flow. The latter is by definition the measure of the total power of electromagnetic radiation emitted or received by a real or virtual surface. We assume that c is an electromagnetic radiation. We will also assume to obtain the dimension $[\text{L}^{-2}]$ of the cosmological constant Λ , that the "1" in the numerator of Eq.23 is of dimension $[\text{M T}^{-3}]$, i.e. kg s^{-3} . This is the dimension of a power surface density in electromagnetism. The author admits that this approach to obtaining the dimension of Λ from $PM_{H_0}^\pm$ is most certainly wrong, but the value of keeping the numerical value of Λ approximated in this way for its dimension seems to him to be most important at this time. This approximate solution brings in his eyes the advantage of consistently solving many open questions in the Λ CDM model by leaving only one open question in this alternative cosmological model.

Note that :

$$\frac{1}{PM_{H_0}^\pm} = \frac{4}{P_{Pl}} \quad (\text{Equation 24})$$

where P_{Pl} is the Planck power.

C.3) Validation of the value of the proposed cosmological constant.

The density parameter of the cosmological constant Ω_Λ in the Λ CDM model is defined by Friedmann equation for a flat universe as follows:

$$\Omega_{\Lambda} = \frac{c^2 \Lambda}{3H_0^2} \quad (\text{Equation 25})$$

i.e . with Planck 2018 results ($H_0 = 67,4 \text{ km/s/Mpc}^{[1]}$, $t_{H_0} = 4,578 \cdot 10^{17}$ seconds today) and the proposed value of Λ :

$$\Omega_{\Lambda} = \frac{299792458^2 \cdot 1,1025 \cdot 10^{-52} \cdot (4,578 \cdot 10^{17})^2}{3} \quad (\text{Equation 26})$$

$$\Omega_{\Lambda} = 0,6923 \quad (\text{Equation 27})$$

By simplifying, today, the matter density parameter $\Omega_m = 1 - \Omega_{\Lambda}$, i.e. $\Omega_m = 0,3077$.

Planck 2018 results^[1] gives $\Omega_m = 0.315 \pm 0.007$. If $\Omega_m = 0.315 - 0.007$, then $\Omega_m = 0.3080$. The theoretical value of Λ gives a result extremely close to the lower bound of Ω_m with the Planck 2018 results^[1]. This is the main reason why the author thinks that the important open question about the determination of the dimension of Λ seems acceptable to him. This alternative cosmological model would give the origin of dark energy where the Λ CDM model fails.

C.4) Proposed solution of the vacuum catastrophe in this alternative cosmological model.

Let us consider the force that attracts our two Hubble mini-spheres $M_{H_0}^+$ and $M_{H_0}^-$ at a distance R_{H_0} . At the point of origin of figure.1, this force crosses at speed c a quantum surface of Planck scale l_{pl}^2 , where l_{pl} is the Planck length. The power $PM_{H_0}^{\pm}$ or energy flux of the cosmological constant thus crosses orthogonally the virtual surface l_{pl}^2 . Mathematically, this gives us :

$$\varphi = \frac{PM_H^{\pm}}{l_{pl}^2} \approx 3,5 \cdot 10^{121} \text{ kg.s}^{-3} \quad (\text{Equation 28})$$

The dimension of is that of a surface power density, i.e., that of the energy flux $PM_{H_0}^{\pm}$ that starts from the origin of the Hubble sphere to interact with its surface. This kg.s^{-3} dimension is also the one that was missing from the numerator when we determined the value of the cosmological constant Λ in Eq.23. And l_{pl}^{-2} is the assumed value of quantum energy suggested by quantum field theory with a cutoff at l_{pl} . One writing of the vacuum catastrophe is divide the energy of the cosmological constant Λ by the vacuum energy suggested by quantum field theory is :

$$\frac{\Lambda}{l_{pl}^{-2}} \quad (\text{Equation 29})$$

Considering the zero point energy suggested by the quantum field theory l_{pl}^{-2} as the inverse of a surface and not as an energy should fit easily into the mirror theories that follow from Andrei Sakharov's hypothesis.

D) Proposal of explanation of the disappearance of antimatter in the Big Bang model.

To make this proposal we must refer to figure 1. To facilitate the understanding of what follows without having to navigate in this file, I make a copy below

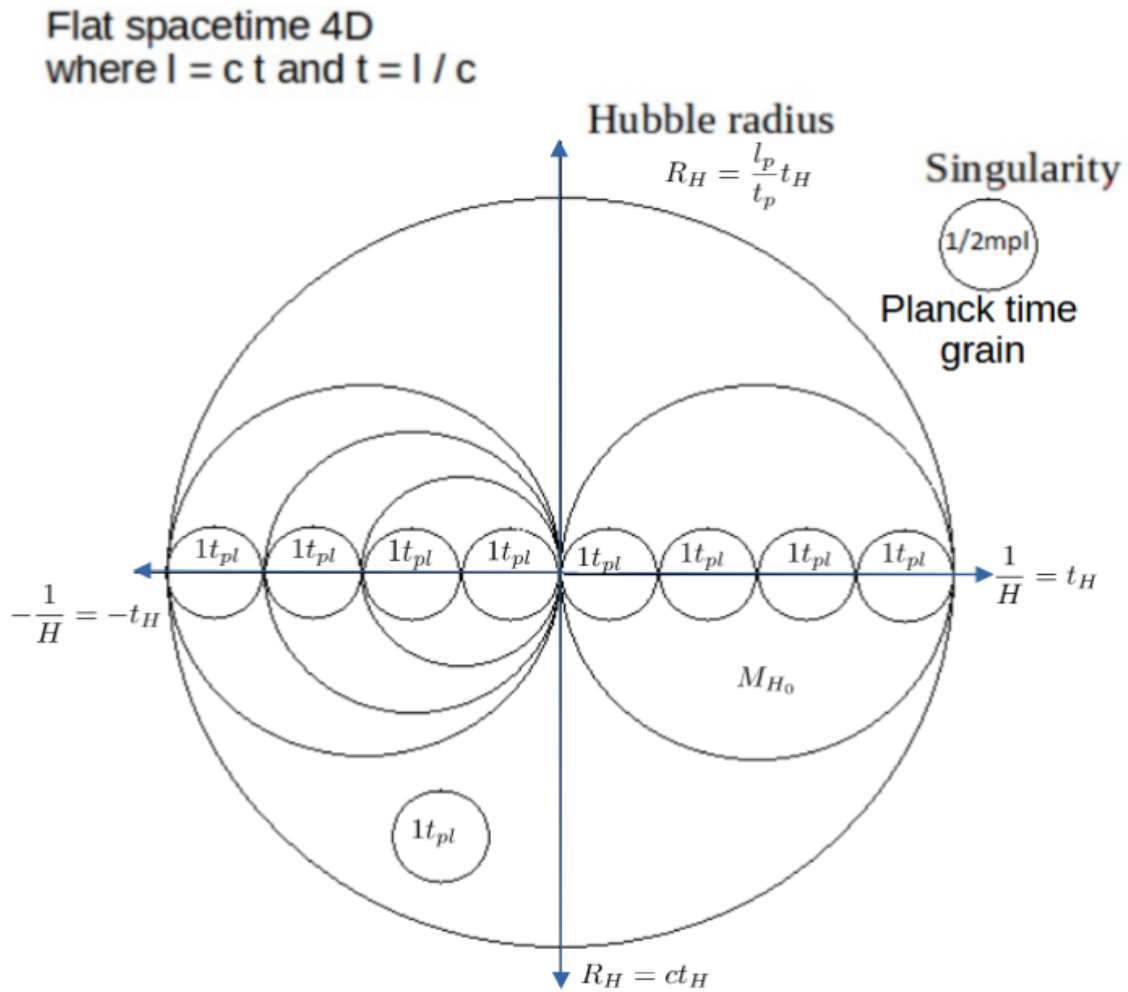


Figure: Hubble sphere

The resolution of this problem comes naturally when the human or instrumental observational bias is identified:

When one observes the Hubble sphere in the R_H "up" direction or in the R_H "down" direction, the two masses M_{H_0} of the two Hubble mini-spheres are not included in the field of view. They are there but the observer placed at the origin of the 4 directions of the figure does not see them. One of these mini-spheres is composed of matter $M_{H_0}^+$, the other of antimatter $M_{H_0}^-$. This is Schrödinger's cat which is both dead and alive as long as we look in the R_H directions. The time can pass as much as one wants, as long as the observer does not change direction of observation, the observer does not know if he is going to "see" the matter or the antimatter. So at the same time, the matter cat is both dead and alive. When the observer will choose to carry out the observation by operating a rotation of 90° , he will see a dead or alive cat. This model of universe starting at $t_{H_0} = 0$ because the time t_{H_0} is signed + or -, the observer will see either $M_{H_0}^+/t_{H_0}^+$, i.e. $M+$ matter, or $M_{H_0}^-/t_{H_0}^-$, i.e. $M-$ matter again. As it is allowed to multiply by 2 the Hubble half-spheres of figure 1, at the origin $t_{H_0} = t_{pl}$ of the Big bang, only the $M_{H_0}^+$ is left by running a positive time arrow. This

explains the infinitesimal rate of antimatter in the observed universe. Antimatter is present everywhere in the universe, but at the Planck scale.

E) Proposal of determination of the CMB temperature in this alternative cosmological model.

I had stressed the importance of considering the Hubble sphere as a black hole at the end of paragraph C.1). Here, I will partially repeat the work of the article "A Rotating Model of a Light Speed Expanding Hubble-Hawking Universe" by U. V. Satya Seshavatharam and S. Lakshminarayana[3] because they give, with an approximation that I would not use, the CMB temperature from the Hawking temperature of black holes.

"(3) Following Hawking's formula for the temperature of black holes [26], the current cosmic temperature can be expressed as follows: "

$$T_{H_0} = \frac{\hbar c^3}{8\pi k_B G \sqrt{m_{Pl} M_{H_0}}} \quad (\text{Equation 30})$$

where \hbar is the reduced Planck constant, or Dirac constant and k_B the Boltzmann constant. The Planck 2018 results give a value of $H_0 = (67.40 \pm 0.5) \text{ km s}^{-1} \text{ Mpc}^{-1}$. Taking the lower bound of the confidence index, we obtain $H_0 = 66.90 \text{ km s}^{-1} \text{ Mpc}^{-1}$ i.e. $t_{H_0} = 4,6124 \cdot 10^{17} \text{ s}$. We obtain with Eq.14, $M_{H_0} = 9,310 \cdot 10^{52} \text{ kg}$.

$$T_{H_0} = \frac{1,0545718 \cdot 10^{34} * 299792458^3}{8\pi * 1.380649 \cdot 10^{23} * 6.6743 \cdot 10^{11} \sqrt{2,176434 \cdot 10^{-8} * 9,310 \cdot 10^{52}}} \quad (\text{Equation 31})$$

note: an error in my latex editor makes it impossible to set the Boltzmann constant to the correct power of 10. Please reify as follows for your calculations:
Boltzmann constant = $1.380649 \cdot 10^{-23} \text{ m}^2 \text{ kg s}^{-2} \text{ K}^{-1}$

$$T_{H_0} = 2,7256 \text{ K} \quad (\text{Equation 32})$$

The CMB temperature measured today, i.e. for $z=0$, is :
 $TCMB(z = 0) = 2.72548 \pm 0.00057 \text{ K}^{[3]}$. The upper bound of the uncertainty error is 2.72605 K. This measurement is therefore in perfect agreement with the calculation made by assimilating the Hubble sphere to a black hole and calculating its Hawking temperature.

Conclusion.

In this alternative model, the mass of the Hubble sphere is equal to $M_{H_0} = \sum_{i=0}^{t_H/t_p} \frac{i m_{Pl}}{2}$ and appears as a "stacking" of Planck half masses on a Hubble timeline t_H instead of a density multiplied by a spherical volume in the Λ CDM model. This stacking of masses is compatible with the apparent isotropy and homogeneity of the universe. This model starts at $t_H = 0$, contrary to the Big bang model, and goes until today. It gives results consistent with the observations made with the Planck satellite. He takes up the idea of double universes and sketches out lines of thought on the relationship between the infinitely small of quantum mechanics and the infinitely large treated with

general relativity. If we still hesitate on the value proposed here of the cosmological constant, we can be much more sure of its basis ($M_{H_0}^+$ and $M_{H_0}^-$ with $FM_{H_0}^\pm$) which allow to theorize the measurement of the CMB temperature and of the interest to carry and renew on Andreï Sakarov's hypothesis.

Finally, we note that the idea of Bruno Valeixo Bento and Stav Zalel in their article "If time had no beginning"^[4] seems correct. By linking it to a quantum space, we can assume that multi-universes could exist everywhere in a flat and infinite 4D space-time, as proposed in figure 1 with singularities inside and outside the Hubble sphere. This is true for each unit of Planck time that passes, but also before the Planck time of the Big Bang.

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Author's final note:

So far, attempts to construct an alternative model to the Λ CDM model have not been able to account for the CMB temperature. To be sure, this alternative model is still incomplete. In particular, it lacks explanations of the power spectrum of the CMB polarization and the power spectrum of galaxies. Since this model does not take dark matter into account, the explanation of the power spectrum of galaxies is currently inaccessible. But according to Chat GPT, the power spectrum of the CMB polarization on the other hand could be accessible with this formula :

$$C_l = \frac{2k_B^4 T_{CMB}^4}{c^2 \hbar^3} \frac{\Omega_b^2}{\Omega_m^2} \int_0^\infty \frac{\Delta_{\mathcal{R}}^2(k)}{k^2} j_l^2(kr) dk$$

with :

C_l is the angular power spectrum of the CMB

$\Delta_{\mathcal{R}}(k)$ is the amplitude of the primordial power spectrum of density perturbations

k represents the wave number. It is a measure of the wavelength of the fluctuation in the early universe, where the initial density fluctuations were generated. In the case of the CMB, the wavenumber is often expressed in terms of angular scales, measured in degrees on the celestial sphere.

j_l is the spherical Bessel function

r is the comoving distance at redshift z corresponding to the angular scale

The verifications that I asked the GPT Cat to perform on the numerical application of his formula are obviously still open to question and are eventually to be explored by the scientific community.